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Two-Part Pricing with Costly Arbitrage

Brian McManus*

This paper considers the optimal two-part pricing strategy of a monopolist whose customers collude when they purchase the firm's product. In contrast to the sentiment in the existing price discrimination literature, I find that a monopolist's profit can actually increase when consumers share its good. When transaction costs for collusion are zero the firm can extract the full consumer surplus through two-part prices. When transaction costs are positive or there are a substantial number of consumers without access to resale, the firm may be hurt by arbitrage.

1. Introduction

A standard assumption for models of price discrimination is that consumers are unable to engage in side transactions after they have purchased a firm's product. This assumption generally helps the firm screen its customers on the basis of their observed or unobserved characteristics. Restrictions on arbitrage are appropriate for many markets.[1] In Walter Oi's (1971) classic example of separate amusement park admission and ride prices, it is hard to imagine a situation in which many consumers can use tickets for rides when only one person is admitted to the park. But surely it is possible for two households to consider buying a lawnmower or theater subscription jointly to avoid the expense of each purchasing independently. Since Oi dissected the "Disneyland dilemma," it has been suggested (but never proven) that such cooperation among consumers would diminish the monopolist's ability to win high profit through nonlinear prices. Oi writes:

> A two-part tariff wherein the monopolist exacts a lump sum tax for the right to buy his product can surely increase profits. Yet, this type of pricing is rarely observed. That apparent oversight on the part of the greedy monopolist can partially be explained by the inability to prevent resale. If transaction costs were low, one customer could pay the lump sum tax and purchase large quantities for resale to other consumers. (1971, p. 88)[2]

Similar arguments are presented in Phlips (1983), Tirole (1988), and Wilson (1993).

In this paper I investigate firm profit and social welfare when consumers can engage in side transactions. A simple model is used to demonstrate that there are situations in which the

* Department of Economics, University of Virginia, P.O. Box 400182, Charlottesville, VA 22904-4182, USA. Present address: Olin School of Business, Washington University, One Brookings Drive, St. Louis, MO 63130, USA; E-mail mcmanus@olin.wustl.edu.

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[1] I follow Tirole (1988) in classifying transactions between consumers under price discrimination as arbitrage.

[2] Although consumer resale as described by Oi would be less desirable than two-part pricing in an arbitrage-free market, the firm must be strictly better off with a nonlinear pricing scheme and the possibility of resale than with nondiscriminatory linear pricing. Even with costless resale a firm could collect a small fixed fee from one consumer without decreasing its (monopoly) profits from unit sales. This paper may be read as an argument that the lump-sum charge collected is not necessarily small.

profit of a monopolist that sets two-part prices can *increase* in the presence of post-sale arbitrage among consumers. The analysis below is divided into two scenarios. In the first, two consumers are able to engage in (costly) side transactions and they are the only individuals eligible to purchase from a monopoly firm. The firm is typically able to increase its profit relative to a model without arbitrage, but there are some (fairly limited) circumstances under which the firm is hurt by collusion between consumers. Profit only falls when transaction costs are neither too large nor too small and demand is appropriately behaved. In the second situation a pair of consumers can costlessly share a monopolist's product, but a number of independent consumers also demand the firm's good. Firm profit increases when the size of the population that cannot engage in side transactions is not too large or when the firm would have sold only to high-demand consumers in a market without arbitrage.

Previous research on price discrimination that has included side transactions or consumer coalitions has found that a monopolist's profit is reduced by the possibility of arbitrage. Alger (1999) submits a model in which a firm offers price/quantity bundles to consumers who can buy cooperatively and purchase multiple bundles. Alger finds that the introduction of multiple and joint purchasing increases the ability of consumers to retain surplus (relative to discriminatory pricing without arbitrage), but there are several differences between her model of cooperative purchase and the analysis below. First, Alger uses general nonlinear pricing (i.e., the firm offers a menu of discrete price/quantity bundles) rather than two-part prices. Second, coalition formation is costless when occurring among consumers with identical demand but prohibited otherwise. I specify below that the two consumers who are able to collude have different demand curves and that arbitrage may require the payment of a positive transaction cost. Third, Alger retains individual participation constraints for independent purchase. Any offered bundle that might be shared must leave a consumer with a nonnegative surplus if the bundle is purchased and consumed independently. In this paper I permit the monopolist to offer pricing arrangements that return nonnegative surplus only under joint purchase.

Innes and Sexton (1993, 1994) study situations in which consumers consider forming a group that will replace an established monopolist in the production of a good. The monopolist uses price discrimination to prevent coalition formation among consumers with identical unit demand curves. The firm offers its product to some consumers at a discount so that the number of consumers who would benefit from a coalition is low enough to prevent a coalition from forming at all.

The remainder of this paper proceeds as follows. In section 2 I present the model of consumer demand, firm pricing, and side transactions. The next section contains a review of optimal pricing by a monopolist when arbitrage is prohibited. In section 4 I consider the case of two colluding consumers and variable transaction costs. The analysis in section 5 covers a firm's pricing strategy when only a subset of the consumer population may engage in arbitrage. Conclusions and extensions to this research are discussed in section 6.

2. The Model

Consider a market in which a single firm produces a good at a constant marginal cost, c. The firm may charge a separate per-unit price (p) and a fixed tariff (F) to capture profit. The timing of pricing and purchasing behavior is as follows. Aware of the composition of the consumer population and the prospects for arbitrage within it, the firm announces a nonlinear price schedule. Any consumer (or group of consumers) that is present in the market is then able to approach the firm and purchase at the posted prices. The firm cannot revise its price schedule between when prices are first announced and when all eligible consumers have had the opportunity to purchase.

All consumers have demand functions that fall into one of two categories. $N + 1$ consumers have demand given by $D_1(p)$, and $N + 1$ consumers have the demand curve $D_2(p)$. The demand functions of the two types of consumer satisfy the following set of assumptions:

ASSUMPTION 0 (A0). Demand has the properties:

(i) D_1 and D_2 are continuous and twice differentiable;

(ii) $D_2(p) > D_1(p) \quad \forall\, p$;

(iii) $D_i'(p) < 0 \qquad \text{for } i = 1, 2;$

(iv) $$-D_i(p) + (p - c)[k_1 D_1''(p) + k_2 D_2''(p)] + 2[k_1 D_1'(p) + k_2 D_2'(p)] \leq 0$$
$$\text{for } i = 1, 2, \quad \text{for any } k_1, k_2 \geq 0, \quad \text{and} \quad \text{for any } p \geq c;$$

(v) Income effects are negligible; and

(vi) If there is a finite p that solves $D_1(p) = 0$, it is greater than c.

Part (ii) of A0 implies that the demand curves do not cross and part (iv) ensures that the firm's profit maximization problem is concave in p for all of the selling strategies discussed below.[3] For convenience, define S_i as the surplus to a consumer of type i from purchasing the efficient quantity at a price $p = c$, i.e., $S_i \equiv \int_c^\infty D_i(p)\, dp$. If a consumer does not purchase from the monopolist, she receives zero surplus.

A single pair of consumers called "consumer 1" and "consumer 2" have the ability to purchase cooperatively. Consumers 1 and 2 belong to the low- and high-demand groups, respectively. This pair of potential buyers pays a nonnegative transaction cost, T, if 1 and 2 cooperate to avoid paying a fixed fee, F, once.[4] The negotiation process between the consumers regarding the payment of positive transaction costs is not explicitly modeled. I assume that consumers only incur the transaction cost if they are able to reach an agreement that increases their joint welfare (measured through consumer surplus) and makes neither consumer worse off. Once a welfare-improving agreement is identified, the consumers are able to buy from the firm and share the purchased good (and its expense) in a way that does not disturb the agreement. Although both the firm and the consumers would bear some expense to change T in certain situations described below, I assume that T is determined exogenously throughout this analysis. Following the terminology of Oi (1971) quoted in the introduction of this paper, side transactions between consumers are sometimes referred to as "resale" of the firm's product.

Social welfare is measured without regard for its distribution among the consumers and the monopolist, that is, welfare is simply the unweighted sum of consumer surplus and firm profit. All comparisons of levels of welfare made in this paper are between two-part pricing schemes with and without resale. Transaction costs are considered to be equivalent to deadweight loss, as T is described as paid out of consumer surplus.

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[3] A0.iv essentially requires that demand is not "too convex." Notice that it is easily satisfied for linear demand.

[4] I may interpret the remaining $2N$ independent consumers as facing prohibitively high transaction costs.

The above assumptions concerning the composition of the consumer population are certainly quite restrictive. One can imagine that a more general model with, say, uncertainty over the demand intensities of consumer coalition members would better represent the decision problem of the firm. Despite this, the results discussed below include a wide range of outcomes.

3. When Arbitrage Is Prohibited

The purpose of this section is to review the firm's problem when arbitrage among consumers is prohibited. Profit maximization leads the firm to choose between two strategies: selling to all consumers (low and high demand) and selling only to consumers with the demand curve D_2. If the firm decides to do the former, it solves the problem

$$\max_p \left\{(2N + 2) \int_p^\infty D_1(s)\, \mathrm{d}s + (p - c)[(N + 1)D_1(p) + (N + 1)D_2(p)]\right\}.$$

The solution of this problem is a price

$$p^* = c + \frac{D_2(p^*) - D_1(p^*)}{-[D_1'(p^*) + D_2'(p^*)]} \tag{1}$$

and a tariff, $F^* = \int_{p^*}^\infty D_1(p)\, \mathrm{d}p$, to be paid by each purchaser. A0.ii, which specifies $D_2(p) > D_1(p)\ \forall\ p$, implies that the second term on the right-hand side of Equation 1 is positive, so $p^* > c$. The monopolist collects

$$\Pi^* = (2N + 2)F^* + (p^* - c)(N + 1)[D_1(p^*) + D_2(p^*)]$$

in profit. The intuition behind why the firm sets p^* above c is rather simple. Suppose the monopolist implemented a pricing policy in which $p = c$ and $F = S_1$. A small increase in p would lead to a second-order decrease in profit from the low-demand consumers, but the firm would enjoy a first-order increase in profit (through unit sales) from high-demand consumers.

When the firm serves both high- and low-demand consumers in the model without arbitrage, it leaves a positive surplus for consumer 2 and there is deadweight loss. At this point it is convenient to define a few terms that will be useful in the next section of this paper. Let A represent the surplus of a high-demand consumer in the standard model. I can write

$$A = \int_{p^*}^\infty D_2(p)\, \mathrm{d}p - \int_{p^*}^\infty D_1(p)\, \mathrm{d}p.$$

Because the firm charges consumers a unit price above marginal cost, deadweight losses arise from sales to each consumer. Let B be the lost surplus under D_1 and let C be the loss under D_2. Exact expressions for the deadweight losses (per consumer) are:

$$B = \int_c^{p^*} D_1(p)\, \mathrm{d}p - (p^* - c)D_1(p^*) \quad \text{and} \quad C = \int_c^{p^*} D_2(p)\, \mathrm{d}p - (p^* - c)D_2(p^*).$$

A, B, and C are depicted on Figure 1. Define $W \equiv A + B + C$, and notice that when $N = 0$, $\Pi^* = S_1 + S_2 - W$.

If the firm decides to sell only to the (identical) high-demand consumers, its profit-maxi-

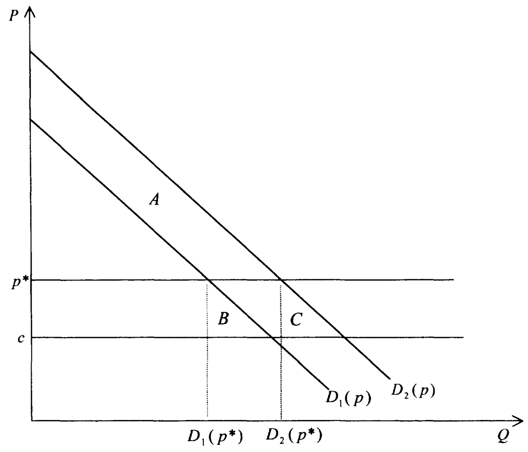

Figure 1. Consumer Surplus (A) and Deadweight Loss (B and C) in the Standard Two-Part Pricing Model

mizing pricing policy is to set a fixed fee equal to S_2 and $p = c$. Profit of $(N + 1)S_2$ is collected. In this situation, an optimal two-part pricing scheme is effectively an instrument of first-degree price discrimination with respect to the high-demand consumers. The monopolist enacts a pricing policy that is efficient with respect to the high-demand consumers, but there are $(N + 1)$ low-demand individuals who do not purchase any of the firm's product. The presence of these consumers results in a deadweight loss of $(N + 1)S_1$.

It is the firm's choice whether to exclude low-demand consumers from purchasing its product. The decision of which pricing plan to enact comes simply from a comparison of Π^* and $(N + 1)S_2$. I offer the following lemma to describe an aspect of this choice and to characterize profit in the standard model:

LEMMA 1: *In the model without resale:*

1. The choice between Π^* and $(N + 1)S_2$ is independent of N, and
2. Maximized profit is continuous and increasing in N, regardless of whether Π^* or $(N + 1)S_2$ is pursued.

PROOF: See the Appendix.

4. Variation in Transaction Costs

In this section I consider the ability of the monopolist to extract surplus from consumers 1 and 2 as T varies. The analysis is split into two cases: when the firm would sell to high- and low-demand consumers when resale is prohibited, and when the firm would only sell to the high-demand consumers. The following assumption holds throughout this section:

ASSUMPTION 1 (A1): $N = 0$ and $T \in [0, \infty)$.

A1 imposes a serious limitation on the population of the market, but it is used to isolate the analysis on variability in transaction costs.

When the Firm Would Serve Both Consumers without Resale

The central additional assumption of this subsection is that consumer 1 has sufficiently strong demand to lead the firm to serve both consumers in the standard model. To this end, I state:

ASSUMPTION 2 (A2): $\Pi^* \geq S_2$.

The analysis that follows from assumptions A0, A1, and A2 is perhaps easiest to digest if it is divided into exhaustive cases for values of T.

Case 1: $T = 0$

When consumers 1 and 2 participate in costless arbitrage, a profit-maximizing monopolist takes the market to an efficient outcome. If the firm sets the unit price, p, equal to marginal cost, it can charge a tariff of $S_1 + S_2$. Both the monopolist and the consumers know that when $T = 0$, the firm will not receive payment of the lump sum tariff, F, more than once. A firm aware of this situation finds the tariff that, when paid only once, maximizes profit while keeping both consumers in the market. The rational monopolist knows that the consumers will cooperate if they have the opportunity to share positive surplus from the monopolist's product. When the only purchasing option available to the consumers is a unit price of c and a tariff that extracts (almost) their entire joint surplus, the consumers purchase $D_1(c) + D_2(c)$ of the firm's product and the firm will collect (almost) $S_1 + S_2$ in profit.

The complete removal of resale costs allows the unit price to equal marginal cost, and there is no deadweight loss. When arbitrage costs among consumers are interpreted as Coasian transaction costs, this result is not surprising. Coase (1960) predicts that well-defined property rights and negligible transaction costs allow economic agents to achieve an efficient allocation of resources. What may be surprising about this result is that I have removed a restriction on consumer behavior, but the firm is able to respond in a way that leaves the consumers worse off.[5] The expansion of a consumer's choice set is usually associated with an increase in her welfare.

Case 2: $T > F^*$

Because this case is fairly simple, I consider it before turning to the more difficult situation in which T takes values between 0 and F^*. The firm's choice of a pricing scheme depends on whether it is advantageous to permit (or induce) resale between consumers 1 and 2. If the monopolist allows its customers to engage in postsale arbitrage when transaction costs are high, the tariff F must satisfy $S_1 + S_2 - T \geq F$. That is, a pricing scheme that results in purchase and resale cannot include a tariff that gives the consumers a negative joint surplus. When $\tilde{F}$ satisfies the condition $\tilde{F} = S_1 + S_2 - T$ and $p = c$, all consumer surplus is accounted for and the firm cannot increase profit while ensuring arbitrage. The monopolist's profit is $\tilde{\Pi} = \tilde{F}$.

Depending on demand conditions and the realized value of T, the firm can sometimes collect strictly more profit than $\tilde{\Pi}$ within Case 2. Recall from section 3 that the profit collected by a monopolist in a model with $N = 0$ and no arbitrage is $\Pi^* = S_1 + S_2 - W$. I have made

[5] Or at least consumer 2 is worse off. In the standard two-part pricing model without arbitrage, the low-demand consumer will not enjoy anything greater than an arbitrarily small amount of surplus. The high-demand consumer collects positive surplus in the standard model; here, she is left with (essentially) zero surplus.

no assumptions that ensure that W is always greater or less than F^*. If $W > F^*$, the firm prefers $\tilde{\Pi}$ to Π^* provided $W > T$. The firm is able to implement the pricing policy designed to return $\tilde{\Pi}$ because the fixed fee charged for this policy, $\tilde{F}$, is only paid once and it leaves the consumers with nonnegative joint surplus. The other possibility for the situation $W > F^*$ is that $T \geq W$. When transaction costs are weakly greater than W, the firm prefers Π^* to $\tilde{\Pi}$. I can be sure that the firm is able to collect Π^* because consumers' savings from avoiding the lump-sum charge more than once (F^*) are less than the cost of doing so (T) by assumption. In summary, when $W > F^*$ within Case 2 the firm is at least as well off with the possibility of resale among consumers as it was without this possibility.

Now consider the situation $F^* \geq W$ while $T > F^*$. Because transaction costs are greater than W, Π^* must be larger than the profit from any pricing scheme that relies on resale and leaves consumers with nonnegative joint surplus. Arbitrage cannot prevent the implementation of the two-part price schedule that returns Π^*. Again, the consumers *could* avoid paying F^* twice through joint purchase, but the expense of this action is larger than the benefit.

Case 3: $0 < T \leq F^*$

The main implication of the assumption maintained throughout Case 3, $0 < T \leq F^*$, is that resale among consumers is inexpensive enough to prevent the firm from collecting F^* from each consumer. As in Case 2, the firm's preferred pricing strategy (and its profitability) depend crucially on the relative sizes of W and F^*. I begin Case 3 by looking at the situation in which $W > F^*$ ($\geq T$). If the firm sets $p = c$ and $\tilde{F} = S_1 + S_2 - T$, it collects more profit than if resale among consumers is prohibited and the firm receives Π^*. As the fixed fee $\tilde{F}$ is constructed to leave the consumers with nonnegative joint surplus, the firm is able to implement its pricing strategy because the only options for the consumers are zero/negligible surplus from joint purchase and zero surplus from not purchasing at all. Thus when $W > F^*$ in Case 3 the firm strictly benefits from the possibility of consumer resale.

Now suppose that $W \leq F^*$. I begin by considering situations in which at least one of the constraints $W \leq F^*$ and $T \leq F^*$ binds. If $W = F^* = T$ the firm is indifferent between: (i) posting p^* and F^*, and (ii) setting $p = c$ and $F = \tilde{F}$. Consumers are also indifferent between the strategies: (i) each pay F^*, and (ii) pay F^* once and incur T to avoid an additional payment of F^*. The firm is able to implement a pricing strategy that yields exactly as much profit as its optimal strategy when resale is prohibited by assumption. If $T < W = F^*$ the firm cannot set a fixed fee as high as F^* and observe payment of it more than once, but in this situation the firm would not attempt to collect a fixed fee from each consumer. With $T < W$ the firm prefers to set $\tilde{F} = S_1 + S_2 - T$ and induce resale, yielding profit that is greater than Π^*. The firm benefits from the possibility of resale among consumers. When $W < F^* = T$, the firm prefers that consumers purchase separately. The monopolist is able to implement the same pricing scheme as when resale is prohibited by assumption because consumers are indifferent between each paying F^* and incurring the expense of purchasing jointly from the firm.

If $W < T < F^*$ the firm cannot collect as much profit as when arbitrage is assumed away. A profit-maximizing pricing strategy that induces resale between the consumers includes a fixed fee that is less than Π^* because T is larger than W. A strategy that leads to the consumers purchasing separately cannot yield profit as high as Π^* because the lump-sum charges that are part of it must be less than F^*. Thus the firm is adversely affected by resale when $W < T < F^*$. If $W = T < F^*$ the firm is unable to implement a pricing strategy that results in separate purchase and yields profit as high as Π^*, but when the firm induces arbitrage it can set a fixed fee of Π^*. In this situation the firm is just as well off when resale is possible as when it is prohibited by assumption. Finally, consider the situation $0 < T < W$. Although the firm cannot implement a pricing strategy that leads each consumer to purchase separately and pay a fixed fee as large as F^*, the firm can do better than Π^* by inducing resale. An optimally sized fixed fee of $S_1 + S_2 - T$ along with $p = c$ leads to firm profit greater than Π^*.

The results on profit from the three cases analyzed above are summarized in the following proposition:

PROPOSITION 1. Under assumptions A0–A2 a monopolist's profit can increase, decrease, or remain the same when a prohibition of side transactions between consumers is removed. Changes in profit depend on:

1. The transaction cost of arbitrage between consumers, and
2. The relative sizes of F^* and W, where F^* is the optimal fixed fee charged by the firm when resale is prohibited and W is the amount of surplus collected by the high-demand consumer plus deadweight loss when resale is prohibited.

When transaction costs are equal to zero, the firm is able to extract all surplus from the consumers by offering its product with a fixed fee that is as large as the summed surplus of the two consumers at the efficient level of consumption.

If demand conditions imply $W \geq F^*$, profit is monotone in transaction costs. Whenever the firm's best pricing strategy is to encourage resale, profit is at least as high as in the standard model and is strictly decreasing in T. Whenever the firm chooses a fixed fee that will be paid by both consumers, it can charge F^* (to collect Π^*). However, if $F^* > W$, there are levels of transaction costs that lead to profit lower than Π^*. Because profit is greater than Π^* when T is small and profit is equal to Π^* for sufficiently large values of T, the possibility of transaction costs that drive profit below Π^* implies that the monopolist's returns are not monotone in T when $F^* > W$.

COROLLARY 1: If demand conditions imply that $W \geq F^*$, profit is monotone (and decreasing) in transaction costs. If $W < F^*$, profit is not monotone in T.

Under the assumptions of case 1 of this subsection, welfare always increases when resale is introduced to the market. The firm sells (indirectly) to each consumer the amount of its product at which demand equals marginal cost, and there is no deadweight loss in the market. However, there are situations in which the firm chooses to induce resale and welfare falls. If the monopolist sets prices that will result in resale, the relevant comparison for whether profit increases is of T and W, but for welfare evaluations I must compare T and $B + C$. When transaction costs, T, exceed the deadweight loss in the standard model, $B + C$, welfare decreases under resale. Because the firm may choose to induce resale when $T > W$ and it is always true that $W > B + C$, situations in which $T > B + C$ and resale occur cannot be ruled out as impossible under the assumptions of section 4.1. However, if transaction costs are smaller than deadweight loss in the standard model ($B + C \geq T$), total surplus is at least as high as when arbitrage is prohibited.

Next, consider situations in which the firm prefers to prevent side transactions between consumers 1 and 2. If the firm is able to announce prices p^* and F^* to collect Π^*, there is no change in total welfare or its distribution. If $F^* > T > W$ and the firm decides to block resale rather than enact a policy that induces arbitrage, it must reduce F^* to make the payment of T

unattractive. When I compare unit prices in monopoly models with and without lump-sum charges, I find that variable prices are lower when two-part pricing is permitted. This is because the monopolist is willing to reduce unit price (and profit on unit sales) to collect profit through fixed tariffs. If $F^* > T$ and a firm that wants to prevent resale cannot charge a tariff as large as F^*, then the firm will not set unit prices as low as p^*. Unit prices higher than p^* increase deadweight loss relative to the model without resale, and total welfare falls. The above analysis of changes in welfare is summarized in the following proposition:

PROPOSITION 2. Under assumptions A0–A2, social welfare can increase, decrease, or remain the same when a prohibition of side transactions between consumers is removed. If T is less than the amount of deadweight loss in the standard model, welfare increases. If $T > B + C$ and the firm induces resale, welfare decreases; if $T > B + C$ and the firm prevents arbitrage, welfare can decrease or remain unchanged.

Unlike profit, welfare is *never* monotone in transaction costs. When T is small enough to lead a monopolist to induce resale by consumers, the firm is the only party in the market that collects surplus. Profit and social welfare decrease as T increases and arbitrage occurs. But when transaction costs become high enough for the firm to switch its pricing strategy from one that encourages arbitrage to one that prevents it, welfare increases abruptly. Consumer 2, the high-demand individual, receives positive (and nonnegligible) surplus. If demand conditions are such that the firm must set a fixed fee less than F^* while discouraging resale, welfare continues to rise as T increases. This further increase in welfare occurs as the firm reduces its unit price toward p^*.

COROLLARY 2: Welfare is not monotone in transaction costs. Welfare decreases in T while the firm chooses to induce resale. Once T is large enough for the firm to ensure that each consumer purchases separately, welfare jumps up and either increases or remains constant in T.

I conclude this subsection with a pair of examples in which (i) there is a range of transaction costs under which profit can fall relative to the standard model, and (ii) firm profit is never reduced by costly arbitrage. In both examples I assume that $c = 0$.

Example 1

Assume $D_1(p) = 13 - p$ and $D_2(p) = 15 - p$. In a market without arbitrage, the prices offered by the firm are $p^* = 1$ and $F^* = 72$. The amount of (potential) consumer surplus not captured by the firm is $W = 27$. Clearly it is possible to have values of T that satisfy $F^* > T > W$, so costly resale can make the monopolist strictly worse off than when arbitrage is prohibited. Figure 2 depicts the profit of a firm operating under the assumptions of Example 1. Profit is not monotone in transaction costs. Profit and welfare are at their lowest when the firm is just indifferent between using a marketing strategy that anticipates consumer cooperation and one that is designed to prevent resale. A small increase in transaction costs from this indifference point results in a small change in firm profit but a substantial increase in social welfare (depicted in Figure 3). The discontinuity in social welfare arises because a firm that sets a fixed fee low enough to prevent resale allows the high-demand consumer to collect a nonnegligible amount of surplus. No consumer receives surplus when the firm executes a pricing strategy that leads to resale.

Example 2

Assume $D_1(p) = 11 - p$ and $D_2(p) = 15 - p$. In the standard model, the prices offered by the firm are $p^* = 2$ and $F^* = 40.5$. Uncaptured consumer surplus is $W = 48$, so there are

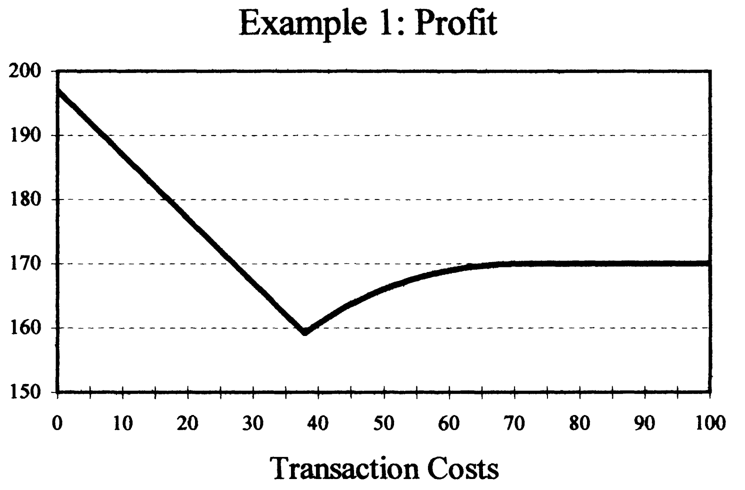

Figure 2. Firm Profit under the Assumptions of Example 1

no values of T that satisfy $F^* > T > W$. For any transaction cost the consumers' ability to participate in arbitrage cannot make the monopolist strictly worse off than when arbitrage is prohibited. Although this implies that profit is monotone in transaction costs (as depicted in Figure 4), welfare jumps up when the firm switches from a pricing policy that encourages resale to one that prevents consumer cooperation. Social welfare is plotted against transaction costs in Figure 5.

When the Firm Would Not Serve Low-Demand Consumers without Resale

In this subsection I reverse assumption A2 and consider the situation in which a monopolist would not serve consumer 1 in a market without resale. This is presented formally as:

ASSUMPTION 3 (A3): $\Pi^* < S_2$.

When this is the case and arbitrage is forbidden, the optimal pricing policy of the firm is

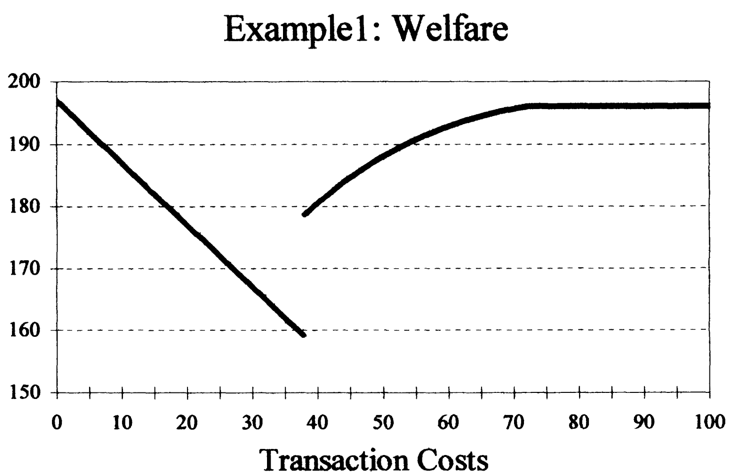

Figure 3. Social Welfare under the Assumptions of Example 1

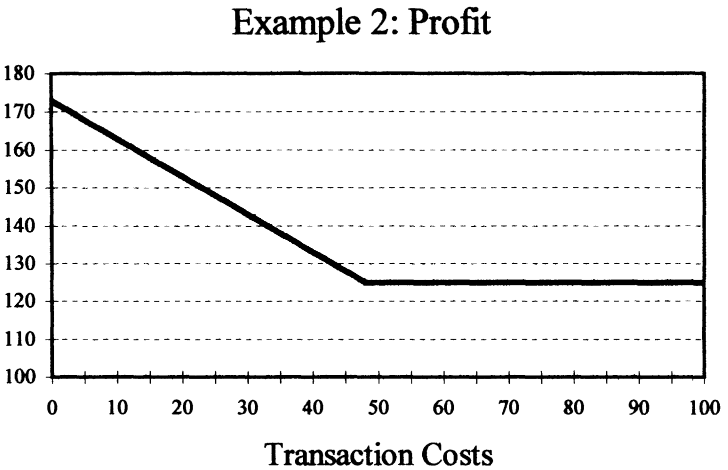

Figure 4. Firm Profit under the Assumptions of Example 2

to set the unit price equal to marginal cost and the lump-sum fee equal to S_2. For the remainder of this subsection I assume that A0, A1, and A3 hold.

Unlike the analysis in the previous subsection, the implications of A3 are rather straightforward. If transaction costs between consumers are zero, the firm can offer a two-part price schedule of $p = c$ and $F = S_1 + S_2$. Whereas the prohibition of resale leads to consumer 1 being "priced out" of the market because of weak demand, when arbitrage is permitted each consumer that is willing to compensate the firm for production costs is able to do so. The selected pricing scheme extracts all surplus from the consumers at the efficient level of production. Again, two-part pricing under costless resale resembles first-degree price discrimination.

As the cost of resale increases, the firm continues to offer a unit of price of c but the fixed fee is adjusted to $S_1 + S_2 - T$. As long as T is no larger than S_1, profit is higher than in the standard model. Reduction of the fixed fee continues until T exceeds S_1, at which point the firm elects to set $F = S_2$ and only the high-demand consumer is served. The impact of resale on profit is summarized in the following proposition:

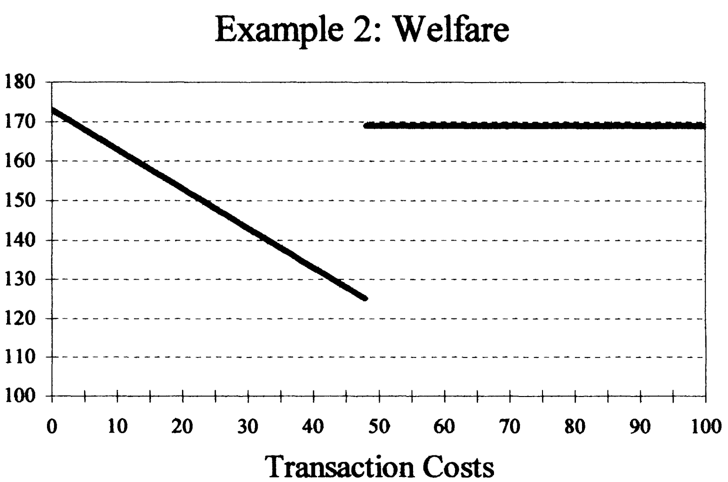

Figure 5. Social Welfare under the Assumptions of Example 2

Proposition 3. Under assumptions A0, A1, and A3 a monopolist's profit is weakly higher when a prohibition of arbitrage between consumers is removed. Specifically, profit is strictly higher when $T \in [0, S_1)$ and profit is unchanged when $T \in [S_1, \infty)$.

Since I know that the firm chooses to induce resale when $T < S_1$ and it just sells to consumer 2 when transaction costs are relatively high, I offer the following corollary:

Corollary 3. Profit is monotonically decreasing in T when A0, A1, and A3 hold. Profit is strictly decreasing for $T \in [0, S_1)$ and it is constant for $T \in [S_1, \infty)$.

Because the firm would have made purchase impossible for the low-demand consumer while resale is prohibited, social welfare is at least as high when consumers can make side transactions. Regardless of whether resale is prohibited, under A3 the profit-maximizing two-part pricing strategy of a monopolist leads to zero surplus among consumers. This means that the increase in profit discussed in Proposition 3 is mirrored by an increase in welfare. If the low-demand consumer is brought into the market through resale and $T < S_1$, social welfare is strictly higher than when arbitrage is prohibited. If transactions among consumers are possible but are relatively expensive ($T \geq S_1$), the price schedule selected by the firm ($p = c$ and $F = S_2$) yields the same amount of total welfare as the standard model.

Proposition 4. Under assumptions A0, A1, and A3 social welfare is weakly higher when a prohibition of arbitrage between consumers is removed. Specifically, welfare is strictly higher when $T \in [0, S_1)$ and welfare is unchanged when $T \in [S_1, \infty)$.

As all consumers receive zero surplus under the assumptions of this subsection, I offer a corollary to Proposition 4 that is very similar to Corollary 3.

Corollary 4. Welfare is monotonically decreasing in T when A0, A1, and A3 hold. Welfare is strictly decreasing for $T \in [0, S_1)$ and it is constant for $T \in [S_1, \infty)$.

5. Heterogeneity in Resale Opportunities

I now describe the optimal pricing policy of the firm when only part of the consumer population can engage in side transactions. I demonstrate below that as $2N$ (the number of consumers who cannot trade the firm's product) grows, the firm adjusts its prices to allow purchases by individuals and not just groups. Section 5 proceeds under A0 and the following assumption:

Assumption 4 (A4): $N \in (0, \infty)$ and $T = 0$.

In A4 I allow N to take any positive real value (not just integers). This eases analysis of how the firm's problem varies with the size of the population unable to engage in resale. If I interpret N as the *portion* of consumers unable to make side transactions rather than a *number* of consumers, this flexibility for values of N is not unreasonable. The assumption that $T = 0$ is made primarily to simplify the analysis below and because positive transaction costs were considered in the previous section.

The remainder of this section is divided into three parts. In the first I characterize the different pricing strategies that might be used by a monopolist under A0 and A4. The next subsection considers the profit and welfare implications of resale when low-demand consumers would be served in the standard model. I then present results on profit and welfare under A0, A4, and the assumption that low-demand consumers would not be served when side transactions are prohibited. As in section 4, I find that the effects of resale on profit and welfare depend on whether the firm would have served the low-demand consumers without arbitrage.

Possible Pricing Strategies under Heterogeneity in Resale Opportunities

As above, the firm is limited to setting one unit price and one fixed fee. There are three general strategies that the firm can use to serve (portions of) the consumer population: (i) Only sell to consumers 1 and 2; (ii) sell to consumers 1, 2, and the N independent high-demand consumers; and (iii) sell to all $(2N + 2)$ consumers. Each of these strategies has a different pair of optimal prices. I denote these prices p_i and F_i, where i corresponds to the list number given above (e.g., p_2 and F_2 are set if the firm decides to serve consumers 1, 2, and the remaining N high-demand consumers). Π_1, Π_2, and Π_3 are the corresponding amounts of profit.

If the firm only serves the consumers who are able to resell its product it sets $p_1 = c$ and $F_1 = S_1 + S_2$, as in section 4. Under this arrangement, profit is $\Pi_1 = S_1 + S_2$. The second strategy that the firm might choose corresponds to a profit maximization problem of

$$\max_{p_2} \left\{ (N + 1) \int_{p_2}^{\infty} D_2(p)\, \mathrm{d}p + (p_2 - c)[D_1(p_2) + (N + 1)D_2(p_2)] \right\}. \tag{2}$$

Note that the firm receives payment of its lump-sum charge $(N + 1)$ times, although there are $(N + 2)$ individuals who consume its product. When $T = 0$ it receives payment from consumers 1 and 2 only once. The solution to Equation 2 is a price determined implicitly by

$$p_2 = c + \frac{D_1(p_2)}{-[D_1'(p_2) + (N + 1)D_2'(p_2)]}. \tag{3}$$

F_2 is set equal to $\int_{p_2}^{\infty} D_2(p)\, \mathrm{d}p$. Although D_1 has a relatively small role in determining p_2, its presence means that the surplus that would be retained by consumers 1 and 2 if the firm set $p_2 = c$ attracts a unit price above marginal cost. Π_2 is the value of the objective function in Equation 2 evaluated at the solution for p_2. If the monopolist chooses to serve all of the consumers in the market (option 3 in the list above), it solves the problem

$$\max_{p_3} \left\{ (2N + 1) \int_{p_3}^{\infty} D_1(p)\, \mathrm{d}p + (p_3 - c)(N + 1)[D_1(p_3) + D_2(p_3)] \right\}. \tag{4}$$

The solution to the problem is a unit price determined by

$$p_3 = c + \frac{(N + 1)D_2(p_3) - ND_1(p_3)}{-(N + 1)[D_1'(p_3) + D_2'(p_3)]}. \tag{5}$$

This price is used in $F_3 = \int_{p_3}^{\infty} D_1(p)\, \mathrm{d}p$, to determine the fixed fee for this marketing strategy. The monopolist collects profit equal to the objective function in Equation 4 evaluated at p_3.

Although it is not necessarily easy to compare profit from the three marketing schemes described above, we establish a pair of useful lower bounds on Π_2 and Π_3.

LEMMA 2. Π_2 is at least at large as $(N + 1)S_2$ and Π_3 is at least as large as $(2N + 1)S_1$.

PROOF: If the monopolist was to set p_2 and p_3 equal to marginal cost, it would collect $(N + 1)S_2$ and $(2N + 1)S_1$ in profit, respectively, from marketing strategies 2 and 3 described above. But if the firm sets p_2 and p_3 according to Equations 3 and 5, the resulting profit would be from $(N + 1)S_2$ and $(2N + 1)S_1$, respectively. *QED.*

The firm can simply compare the profit from each of these marketing strategies to decide which has the greatest return for the observed demand conditions. The firm will move among the strategies in a fairly reasonable way; this is reflected in the following lemma.

LEMMA 3. For sufficiently high values of N, $\Pi_1 \neq \max\{\Pi_1, \Pi_2, \Pi_3\}$.

PROOF: Π_1 is always $(S_1 + S_2)$, whereas $\Pi_2 \geq (N + 1)S_2$ and $\Pi_3 \geq (2N + 1)S_1$. Since these lower bounds on Π_2 and Π_3 are increasing in N, it must be the case that for sufficiently high values of N $\Pi_1 \neq \max\{\Pi_1, \Pi_2, \Pi_3\}$. *QED.*

The intuition behind Lemma 3 is simply that as the size of the population without recourse to arbitrage grows, the firm eventually finds it worthwhile to set prices in a way that allows independent consumers to purchase.

When High- and Low-Demand Consumers Would Be Served without Resale

If demand is such that both high- and low-demand consumers are served without resale, the firm either can benefit or have its profit reduced by the introduction of costless side transactions. This result is presented in the following proposition:

PROPOSITION 5. Assume that the monopolist would serve low-demand consumers when resale is prohibited: $\Pi^* > S_2$. Under this assumption, A0, and A4, the introduction of resale between consumers 1 and 2 can either increase or decrease the firm's profit.

PROOF. See the Appendix.

In light of the analysis in section 4, the reasoning behind possible increases in profit should be straightforward. If N is small enough, the situation described in Proposition 5 is similar to when N and T are zero and the firm's profit strictly increases with the introduction of resale between consumers 1 and 2. With a small, positive value of N the monopolist may choose to forgo the opportunity to sell to the $2N$ independent consumers in the market and instead focus on extracting maximal surplus from consumers 1 and 2. The possible reduction in profit mentioned in Proposition 5 essentially arises when the population of independent consumers becomes too large to ignore but the firm cannot serve these individuals without foregoing a relatively substantial amount of surplus to the consumers who can engage in side transactions. Because consumers 1 and 2 would not have been in a position to retain as much surplus without the possibility of resale, this leads to a reduction in profit.

Introducing side transactions can lead to a change in the composition of the active consumer population. The firm may exclude all $2N$ independent consumers or just the N independent low-demand consumers.

PROPOSITION 6. Assume that the monopolist would serve low-demand consumers when resale is prohibited: $\Pi^* > S_2$. Under this assumption, A0, and A4, the introduction of arbitrage between consumers 1 and 2 can lead the firm to pursue Π_1, Π_2, or Π_3.

PROOF: See the Appendix.

The intuition behind the possibility of Π_1 being the largest or the smallest return was presented in Proposition 5 and Lemma 3, respectively. Perhaps the more interesting part of this proposition is that the firm's choice to serve both high- and low-demand consumers when resale is prohibited does not necessarily mean that the firm will always find $\Pi_3 \geq \Pi_2$. The reason behind this is something subtler than the fact that consumer 1 moves out of the population of low-demand consumers once resale is possible. The relative number of independent high- and low-demand consumers is the same as in the standard model. However, when the N low-demand consumers are brought into the market, the firm must reduce the amount of profit that it takes from the N independent high-demand consumers *and* consumers 1 and 2.

Just as profit can increase or decrease with the introduction of resale when all consumers would be served without arbitrage, the change in overall social welfare will not always have the same sign.

PROPOSITION 7. Assume that the monopolist would serve low-demand consumers when arbitrage is prohibited: $\Pi^* > S_2$. Under this assumption, A0, and A4, introducing resale between consumers 1 and 2 can either increase or decrease social welfare.

PROOF: See the Appendix.

When N is very small, welfare always increases with the introduction of resale between consumers 1 and 2. The reasoning behind this is essentially the same as the argument that profit always increases with resale when N is close to zero. When the only consumers in the market are 1 and 2 and $T = 0$, welfare is strictly higher when resale is possible. Like profit, welfare in the standard model is continuous in N, so a small increase in N away from zero will not change the welfare ranking between the situations with and without resale for $N = 0$.

Welfare always decreases with resale when the firm chooses to serve all $(2N + 2)$ consumers in the market. Deadweight loss is positive in the standard model because the firm benefits from setting its unit price above marginal cost when serving (equal numbers of) high- and low-demand consumers. If resale is possible and the monopolist pursues Π_3, there is still an equal number of independent high- and low-demand consumers, but now there is also a group of buyers that will receive a larger rent than the N high-demand individuals: consumers 1 and 2. The prospect of collecting additional profit from consumers 1 and 2 through unit sales leads the firm to install an additional increase of price above marginal cost.

When Low-Demand Consumers Would Not Be Served without Resale

Having found that profit can either increase or decrease with resale when the firm would have served all consumers, I now consider the situation in which the firm would *not* have served all consumers when side transactions are prohibited. As in section 4, I find that permitting arbitrage between consumers 1 and 2 always leads to (weakly) higher profit than the standard model. If the firm finds that $\Pi_1 = \max\{\Pi_1, \Pi_2, \Pi_3\}$, it can set its prices so that it sells only to consumers 1 and 2. If the number of independent consumers is too large for the firm to ignore, the monopolist can sell to all of its customers when resale is prohibited (the high-demand group) plus one low-demand individual (consumer 1). The firm does not have to sacrifice any of the profit it takes from the independent high-demand consumers to serve consumer 1. These results are presented in the following proposition:

PROPOSITION 8. Assume that the monopolist would not sell to low-demand consumers when arbitrage is prohibited: $\Pi^* \leq S_2$. Under this assumption, A0, and A4: (i) introducing resale between consumers 1 and 2 cannot reduce firm profit; (ii) for a sufficiently large N, $\Pi_2 > \Pi_1$, and (iii) for any $N > 0$, $\Pi_2 > \Pi_3$.

PROOF. See the Appendix.

The third listed result in Proposition 7 implies that the firm never serves low-demand individuals other than consumer 1 under the assumptions of this subsection. Resale does not change the way the firm regards low-demand consumers that are prohibited from arbitrage. Contrast this with Proposition 6, which establishes that introducing resale between consumers 1 and 2 can lead the firm to choose the pricing strategies that yield Π_1, Π_2, or Π_3.

The fact that profit is always higher with resale under the assumptions of this subsection leads directly to a welfare result. When side transactions between consumers are prohibited, the total amount of welfare in the market is the profit generated by sales to independent high-demand consumers. But social welfare is at least as high as profit. If resale between consumers 1 and 2 leads to higher profit then it must also lead to higher welfare. Although this welfare comparison is true for all N, welfare under resale jumps up when the monopolist switches from pursuing Π_1 to Π_2. When the firm sells to the N independent high-demand individuals, consumers 1 and 2 each receive a positive amount of surplus and social welfare strictly exceeds profit. This result is summarized (without proof) in the following proposition:

PROPOSITION 9. Assume that the monopolist would not sell to low-demand consumers when arbitrage is prohibited: $\Pi^* \leq S_2$. Under this assumption, A0, and A4, welfare is always higher when resale between consumers 1 and 2 is permitted.

6. Conclusion

In this paper I have argued that a monopolist can be made better off by the presence of postsale arbitrage among consumers. In the most extreme situation (when all consumers have access to costless resale), the increase in monopoly profit is accompanied by an efficient allocation of resources. However, there are situations in which the firm is adversely affected by resale. In the case of costly arbitrage between two consumers (section 4), it is possible to observe a range of transaction costs and demand conditions under which firm profit is lower. This result reinforces the conventional view that monopolists are hurt by arbitrage, but not as the literature suggests—where transaction costs are zero or very low. A consumer population that is mixed in its access to resale can also lead to lower profit.

The ability of the firm to increase profit is not driven by the assumption of two-part pricing. For example, if a firm sells discrete price/quantity bundles in a situation without resale between consumers with different demand intensities, it preserves a rent for the high-demand buyer. But when the same firm faces consumers with the ability to engage in costless resale, it will sell one large bundle that extracts (almost) all consumer surplus. The crucial assumption in this paper is that of arbitrage between consumers with different demand intensities. The firm is able to benefit from cooperation among buyers because resale allows the firm to capture the surplus otherwise received by a high-demand consumer. This interaction of heterogeneous buyers is not troubling if I believe that a consumer's likely purchasing partners (perhaps neighbors, friends, or a spouse) can have intensities of demand that do not match her own.

Simultaneously selected two-part price schedules in a homogeneous-good oligopoly market would not include the features of interest described above; in equilibrium all firms would set $F = 0$ and $p = c$. However, in alternative modeling frameworks, positive fixed fees and unit prices above cost may be observed. Heywood and Pal (1993) find that the sequential selection of two-part prices in a homogeneous-good duopoly can lead to positive fixed fees and margins on unit sales. There is also a growing literature on nonlinear prices in differentiated-product oligopolies in which the local market power of firms permits some discretion in pricing strategies. See Wilson (1993) or Stole (1995) for examples.

I used a simple model to study pricing and profit under arbitrage with minimal technical distractions. There are several directions in which the analysis can be extended. Among the topics for further study are: (i) a richer model of uncertainty over consumer demand intensities, (ii) an explicit model of consumer interaction, (iii) the introduction of intermediaries or brokers who facilitate cooperation among consumers, and (iv) the description of transaction costs as sensitive to the number of agents interacting and to the efforts of agents to alter these costs. The relevance of the pricing strategies (and profit and welfare results) described here will be strengthened if these strategies also emerge in models that include the extensions listed above.

There are many markets to which I can look for examples of consumer resale or sharing. Neighbors may make a joint purchase of a lawnmower or snowblower. Sports fans may divide a season ticket among several people. Vacationers who cannot afford holiday homes may join a time-share association. Shoppers may share payment of a sign-up fee to a members-only discount store. New theoretical studies that address the pricing and purchasing phenomena in these markets should attempt to explain why resale or cooperative purchase is sometimes tolerated and sometimes discouraged by firms.

Appendix

LEMMA 1. In the model without resale: (i) the choice between Π^* and $(N + 1)S_2$ is independent of N, and (ii) maximized profit is continuous and increasing in N, regardless of whether Π^* or $(N + 1)S_2$ is pursued.

PROOF: The first part of this lemma is established through a comparison of Π^* and $(N + 1)S_2$. Because p^* and F^* are independent of N, I can factor $(N + 1)$ out of Π^* to separate any influence of population size on this measure of profit. Write $(N + 1)\pi^* = \Pi^*$. Any comparison of Π^* and $(N + 1)S_2$ turns on the relative sizes of π^* and S_2, which are both independent of N.

The second part of the lemma follows simply from the fact that π^* and S_2 are independent of N. Because $(N + 1)$ is continuous and increasing in N, $(N + 1)$ multiplied by either π^* or S_2 must also be continuous and increasing in N. *QED.*

PROPOSITION 5. Assume that the monopolist would serve low-demand consumers when resale is prohibited: $\Pi^* > S_2$. Under this assumption, A0, and A4, the introduction of resale between consumers 1 and 2 can either increase or decrease the firm's profit.

PROOF. First I show that profit can be greater under resale, and then we show that it can be lower.

Greater: In section 4 we established that when $N = 0$ and $T = 0$, firm profit under resale is strictly higher than without resale. The amount of profit collected in this situation is $S_1 + S_2$, which is equal to Π_1 and is independent of N. Lemma 1 establishes that maximized profit without resale is continuous in N, so there must exist small, positive values of N for which profit with resale is higher.

Lower: Lemma 3 states that when N is sufficiently large, $\Pi_1 \neq \max\{\Pi_1, \Pi_2, \Pi_3\}$. Let $\hat{N}$ be a value of N for which $\Pi_1 \neq \max\{\Pi_1, \Pi_2, \Pi_3\}$. Next, note that as N increases, $p_2 \to c$ and $\Pi_2 \to (N + 1)S_2$. An assumption of Proposition 5 is that $\Pi^* > (N + 1)S_2$, and the convergence of Π_2 to $(N + 1)S_2$ means that for large enough values of N, $\Pi^* > \Pi_2$. Let $\bar{N}$ be a value of N that satisfies this inequality, and define $N^* = \max\{\hat{N}, \bar{N}\}$. Thus $\Pi^* > \Pi_2$ for $N = N^*$. Finally, I show that $\Pi^* > \Pi_3$ when $N = N^*$. Let Π_{S3} be the amount of profit the firm would collect in the standard model (without resale) when it chooses p_3 and F_3. It must be true that $\Pi_{S3} \leq \Pi^*$. A comparison of Π_{S3} and Π_3 reveals $\Pi_{S3} - F_3 = \Pi_3$, as the firm sells the same amount of its product (to the same consumers) but receives its fixed fee one less time. $\Pi^* \geq \Pi_{S3} > \Pi_3$ for $N = N^*$. Therefore at N^*, Π^* is larger than the profit from any pricing strategy from the firm under resale. *QED.*

PROPOSITION 6. Assume that the monopolist would serve low-demand consumers when arbitrage is prohibited: $\Pi^* > S_2$. Under this assumption, A0, and A4, the introduction of the possibility of resale between consumers 1 and 2 can lead the firm to pursue Π_1, Π_2, or Π_3.

PROOF. The three selling strategies are established in turn.

Select Π_1: Π_1 is strictly greater than Π_2 and Π_3 when $N = 0$, so there must also be small, positive values of N for which Π_1 is preferred.

Select Π_2: Under some parameterizations of the model, Π_2 is dominated by either Π_1 or Π_3 for all N. But there are functional forms for which $\Pi_2 = \max\{\Pi_1, \Pi_2, \Pi_3\}$ holds for some N. A simple example of such a case is when $D_1(p) = 10 - p$, $D_2(p) = 13 - p$, $c = 0$, and $N = 1$.

Select Π_3: Suppose N is large enough to yield $\Pi_3 > \Pi_1$. Now consider the relative sizes of Π_2 and Π_3 as N increases further. Because $p_3 \to p^*$ as N increases, $\Pi_3 \to (\Pi^* - F^*)$. Because $p_2 \to c$ as N grows, $\Pi_2 \to (N + 1)S_2$. Next, recall from Lemma 1 that I can write $\Pi^* = (N + 1)\pi^*$. Divide through $\Pi_3 \to (\Pi^* - F^*)$ by $(N + 1)$ to obtain $\Pi_3/(N + 1) \to [\pi^* - F^*/(N + 1)]$. The effect of the $F^*/(N + 1)$ term disappears as N grows, and I find that $\Pi_3/(N + 1)$ approaches π^* and $\Pi_2/(N + 1)$ goes to S_2 as N increases. Because I have assumed $\Pi^* > (N + 1)S_2$ and I know from Lemma 1 that $\pi^* > S_2 \leftrightarrow \Pi^* > (N + 1)S_2$, I can conclude that $\Pi_3 > \Pi_2$ for a large enough N. *QED.*

PROPOSITION 7. Assume that the monopolist would serve low-demand consumers when arbitrage is prohibited: $\Pi^* > S_2$. Under this assumption, A0, and A4, the introduction of the possibility of resale between consumers 1 and 2 can either increase or decrease social welfare.

PROOF: First I show that welfare can be greater under resale, and then I show that it can fall.

Greater: An increase in welfare when $T = 0$ and $N = 0$ was established in section 4. Because welfare is in continuous in N, this increase in welfare must be maintained for (at least) some small, positive values of N.

Lower: The proof of Proposition 6 established that the firm chooses Π_3 when resale is permitted and N is sufficiently large. If $p_3 > p^* \; \forall \; N$, welfare is greater in the standard model than under resale for values of N high enough to lead to Π_3. I can write Equation 5 as

$$p_3 = c + \frac{D_2(p_3) - D_1(p_3)}{-[D_1'(p_3) + D_2'(p_3)]} + \frac{D_1(p_3)}{-(N + 1)[D_1'(p_3) + D_2'(p_3)]}. \tag{A1}$$

Assumption A0.iv ensures that the solutions for all unit prices derived in this paper are unique. Equation A1 only differs from the expression that sets p^* in the last term on the right-hand side. Since this part of Equation A1 is strictly positive by assumption, it must be the case that p_3 is always larger than p^*. As deadweight loss is higher when all consumers are served under resale (relative to the standard model), social welfare must be lower. *QED.*

PROPOSITION 8. Assume that the monopolist would not sell to low-demand consumers when arbitrage is prohibited: $\Pi^* \leq S_2$. Under this assumption, A0, and A4, (i) introducing resale between consumers 1 and 2 increases firm profit; (ii) for a sufficiently large N, $\Pi_2 > \Pi_1$; and (iii) for any $N > 0$, $\Pi_2 > \Pi_3$.

PROOF. To begin the proof of part 1, recall that $\Pi_2 \geq (N + 1)S_2$ from Lemma 2. When resale is prohibited the firm collects $(N + 1)S_2$ in profit. Since Π_2 is greater than profit without resale and $\max\{\Pi_1, \Pi_2, \Pi_3\} \geq \Pi_2$, the firm's preferred marketing strategy when resale is possible must be more profitable than its optimal strategy when resale is prohibited by assumption.

Part 2 follows directly from Lemma 3.

To prove part 3 I only need to note that an assumption of this proposition is that $(N + 1)S_2 \geq \Pi^*$ and that the proofs of Lemma 2 and Proposition 5 establish $\Pi_2 \geq (N + 1)S_2$ and $\Pi^* > \Pi_3$. *QED.*

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